Faster than a Speeding Bullet
The Basics Aerodynamics of Supersonic Flight

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Abstract

This document contains a discussion of the basic principles which create lift, the upward force which maintains an aircraft's flight, in both subsonic and supersonic conditions. Areas of discussion include Bernoulli's Principle, the Continuity Principle, Euler's Equation, Oblique Shock Waves and Expansion Waves, and supersonic aircraft stability problems.

It had scarcely been half a century since man had first conquered the skies, and already he was on a quest to penetrate the sound barrier—unimaginable only a few years prior. As propulsion technologies got more and more advanced, as well as manufacturing methods to produce less equipment failures, the possibility to exceed the rate at which sound travels became somewhat more likely. However, the hurdles that were encountered along the way were created by the remarkable difference in the behavior of a fluid as it reaches or exceeds the speed of sound (or, for that matter, by any object exceeding the speed of sound through it). Some of these differences were explained with science, others through trial and error (through which many lost their lives), but finally, on October 14, 1947, Captain Charles E. Yeager broke the sound barrier in the Bell XS-1 rocket plane (Boyne, 1997). What follows is a discussion of the hurdles those scientists had to overcome on the road to that event.

The first major problem with supersonic flight is that the basic principles of aerodynamics simply do not hold true past the speed of sound, or Mach 1.0. At relatively slow speeds, air is considered to be "incompressible," yielding small changes in density for relatively large pressure changes. David Bernoulli originally described the formula for subsonic flight in the mid–eighteenth century, but its relevance to airfoil design was not put into practical use until, obviously, much later. Bernoulli's theorem was simply a rewording of the familiar law of conservation of energy (which was, however, later disproved by Einstein): The total pressure of a fluid is equal to its "static pressure" added to its "dynamic pressure" (Giancoli, 1995), or, in other words, potential plus kinetic energy adds up to total available energy (neglecting temperature or gravity, among other things). Mathematically, it can be written $P + 1/2\ \rho v2 = k$ where "P" is static or ambient pressure, "$1/2\ \rho v^2$" is dynamic pressure, "k" is a constant, and there exists a consistent effect of gravity (constant height).

Of equal importance to aerodynamics was the continuity equation, which states that an increase in cross sectional area of a tube with fluid of constant density flowing through it will cause a corresponding reduction in velocity of the fluid through the tube. To prove it, let's assume there exists a flow of incompressible air is flowing through a tube at a rate of $^{\Delta m}/_{\Delta t}$. Δm can be represented also by breaking down the constant density and volume through a specific length of the tube (area being constant), so Δm is equal to $\rho \Delta V$. Since cross sectional area is also constant, ΔV is equal to the area of the tube, A, times the length, Δl over which the measurement is taken (which is arbitrary). The velocity of the fluid (v) moving through the tube is $^{\Delta l}/_{\Delta t}$, so the logic concludes as follows:

$$\frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A \Delta l}{\Delta t} = \rho A v$$

The quantity Av is sometimes referred to as the volume flow rate, or "flux", and is a constant for a constant density of a fluid. If the density is included ($\rho A v$), it is referred to as "mass flux" (McCormick, 1979).

Taking the above equation and changing the area will result in a corresponding change in velocity, since in slow speed regimes density of air remains the same. Bernoulli then told us that with an increase in velocity, we have a corresponding decrease in pressure. Cambered airfoils are designed to take advantage of this; as the air strikes the upper and lower surfaces of the airfoil, air is forced upward to a surface of larger area than the lower side, which increases its velocity (from the continuity equation), therefore reducing pressure, creating a pressure gradient and therefore a resulting force F_{press} =

$-\nabla P dV$, producing lift (Boeker & Grondelle, 1995).

Unfortunately, many individuals paid with their lives to discover that air is, in fact, compressible. No longer do the above conclusions necessarily hold true, and so a new branch of aerodynamics was born to deal with these "compressibility effects."

First, let's ensure that all appropriate principles of gas dynamics have been considered. The continuity equation told us that $\rho A v = k$. Differentiating by parts, this equation can be written as follows:

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

Now let's do some magic to Bernoulli's equation. If we take Bernoulli's equation of $P + {}^{1}/_{2}\,\rho v^2 = k$ and differentiate, we get:

$$v\,dv + \frac{dP}{\rho} = 0$$

The above equation is referred to as Euler's Equation, which accommodates for compressible flow (Liepmann & Puckett, 1947). Since all sound waves are pressure and density divergences, and they all travel at the speed of sound (a), it can be shown that dP/dr is equal to the speed of sound, a, squared. This can be substituted into the continuity equation, and then the equations can be simplified to the following:

$$\frac{dv}{v}\left(\frac{v^2}{a^2} - 1\right) - \frac{dA}{A} = 0$$

Since $v^2/_{a}{}^2$ is simply the Mach number squared, the above equation can be reduced to ${}^{dv}/_{v}\,(1- M^2) = -{}^{dA}/_{A}$, which has interesting implications (Liepmann & Puckett, 1947). Previously, it was determined that through a converging duct (which is what a positively cambered airfoil can be thought of as), velocity of the fluid would increase. *However, as the above equation shows, above the speed of sound, converging ducts will slow down air velocity.* Bernoulli's theory, therefore, has failed us (Liepmann & Puckett, 1947).

Yet properly designed aircraft can, and do, fly at and beyond the speed of sound. However, the principles which they utilize are somewhat unique compared to their subsonic counterparts. Unlike flow in subsonic regimes, all changes of velocity, pressure, density, flow direction, etc, take place "quite suddenly and in relatively confined areas" (Hurt, 1965). These areas that are formed are typically referred to as "shock waves," though expansion waves also play a great part in the production of lift, and are not technically shock waves.

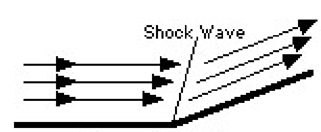

Figure 1: Oblique Shock Wave

Consider Figure 1, above. Here we have a positively cambered deflection of air, like a converging duct. This area would create, as determined in the previous equations, a decrease in velocity, and therefore an increase in static pressure. Figure 2 is the opposite; due to the diverging duct effect, this would cause an increase in velocity with a corresponding decrease in pressure. Once again, differential pressures have created the lift, but in an entirely new way, as shown in Figure 3. The expansion waves have created the positive upward forces, but this only works in a positive angle of attack.

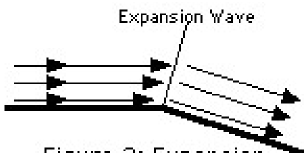

Figure 2: Expansion Wave

Not only do engineers need to compensate for the different method of producing lift, but also must deal heavily with stability. During the conversion of subsonic to supersonic flow, the Aerodynamic Center, or point at which the pitching moment is constant throughout all angles of attack (Smith, 1992), moves near the 50% MAC position. Since most airfoils in subsonic flight have the Aerodynamic Center at approximately 25% MAC, this shift in pitching moment can require serious trim adjustments to maintain stable flight charateristics and positive aircraft control (Hurt, 1965).

It was a feat, but mankind succeeded in exceeding the speed of sound, and has since doubled, and even tripled it. Though supersonic commercial aviation is a very limited industry at present, the future will undoubtedly create a demand for such incredibly fast travel, and as airfoil and powerplant technologies become more and more efficient and practical, the pilot of the future will unquestionably find himself at the controls of, as we've seen above, a very complicated machine. It goes without saying then that a basic knowledge of the principles of supersonic aerodynamics will be required to continue the safety record which has already made flying the safest way to "get there." Fortunately, like most other pilots, I look forward to the challenge.

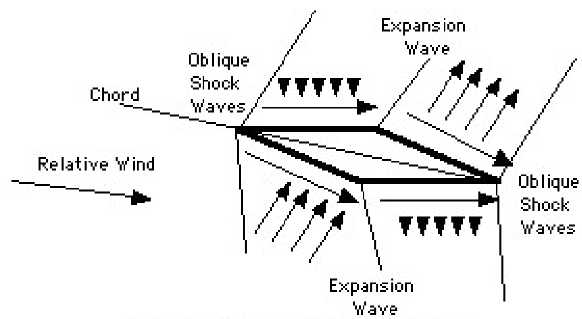

Figure 3: Creation of Lift on a Double Wedge Supersonic Airfoil

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